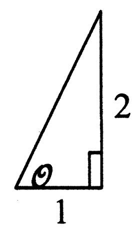
# TRIG & LOGS WITHOUT CALCULATORS

## For Trig Problem Success

1. Know the exact trig values for special angles: 0°, 30°, 45°, 60°, 90°, etc.
2. Know as many trig identities and trig relationships as possible, including co-function and reciprocal relationships; sum, difference and double angle formulas and Pythagorean identities.
3. Know how to use any given trig value (or known trig values) to generate other exact values by placement in a right Δ, and then applying the Pythagorean theorem and SOH-CAH-TOA.

   *For example*: Find cos θ, if tan θ = 2 and $0 \le \theta \le \pi/2$

| | |
|---|---|
| Draw a rt.Δ with tan θ = 2 (See diagram at right.)<br>Next use the Pythagorean Thm to calculate the hypotenuse: $h = \sqrt{1^2 + 2^2} = \sqrt{5}$<br>Thus $\cos\theta = \frac{1}{h} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ |  |

## For Log Problem Success

1. Know basic log rules that can be used to rewrite given expressions, including the change of base rule.
2. Since $\log_b b = 1$, it is sometimes advantageous to rewrite $\log_b N$ in terms of the base *b*.

   *For example:* Simplify $\log_{\sqrt{x}} x^x$

$$\log_{\sqrt{x}} x^x = \log_{\sqrt{x}}\left(\left(\sqrt{x}\right)^2\right)^x = \log_{\sqrt{x}}\left(\sqrt{x}\right)^{2x}$$

$$= 2x\left(\log_{\sqrt{x}} \sqrt{x}\right) = 2x(1)$$

$$= 2x$$

TRIG & LOGS WITHOUT CALCULATORS:   PRACTICE SET

[89i6] 1. Compute the smallest positive angle, in degrees, such that

$$\tan 4x = \frac{\cos x - \sin x}{\cos x + \sin x}$$

[98I6] 2. If $\sec x + \cos x = 3$, compute the value of

$$\frac{\sec^{10} x + 1}{\sec^5 x}$$

[10I8] 3. Compute x, where

$$\log_{1/x}(x) + \log_x(x^5) = \log_{\sqrt{x}}(x^{x^2})$$

[84I2] 4. If $0° < x < 90°$ and $\cos x = \frac{3}{\sqrt{10}}$, compute

$$\log \sin x + \log \cos x + \log \tan x$$

[03I6] 5. Compute the smallest θ > 0, where θ is in radians, for which

$$\sec\theta = \log_{\sqrt{3}}\left(\frac{1}{\log_{\sqrt[3]{\pi}}(\pi)}\right)$$

*Questions on this practice were selected from NYSML competitions, where the year and problem number are indicated in brackets. These are to be used only for practice in preparation for future NYSML or NYSML member competitions.

Answers:

1. 9° 2. 123 3. $\sqrt{2}$ 4. -1 5. $2\pi/3$

Solutions on next page.

1. Given $\tan 4x = \dfrac{\cos x - \sin x}{\cos x + \sin x}$ substitute $\dfrac{\sin 4x}{\cos 4x} = \dfrac{\cos x - \sin x}{\cos x + \sin x}$ and cross-multiply.

So $\sin 4x(\cos x + \sin x) = \cos 4x(\cos x - \sin x)$

$$\sin 4x \cos x + \sin 4x \sin x = \cos 4x \cos x - \cos 4x \sin x$$
$$\sin 4x \cos x + \cos 4x \sin x = \cos 4x \cos x - \sin 4x \sin x$$
$$\sin(4x + x) = \cos(4x + x) \quad \text{so} \quad \sin(5x) = \cos(5x) \quad \text{and} \quad \tan(5x) = 1$$

Since we want the smallest positive angle we know, $5x = 45°$ so $x = 9°$

2. For simplicity, let $\sec x = s$, $\cos x = c$.

We know $sc = 1$ (product of reciprocals) and $s + c = 3$ (given)

$$(s+c)^2 = s^2 + 2sc + c^2 = s^2 + 2 + c^2 = 3^2 \quad \text{so} \quad s^2 + c^2 = 9 - 2 = 7$$
$$(s+c)^3 = s^3 + 3s^2c + 3sc^2 + c^3 = s^3 + 3sc(s+c) + c^3 = 3^3$$
$$\text{so} \quad s^3 + 3(3) + c^3 = 27 \quad \text{or} \quad s^3 + c^3 = 18$$
$$(s^2 + c^2)(s^3 + c^3) = s^5 + s^2c^3 + s^3c^2 + c^5 = (7)(18) = 126$$

But $= s^5 + s^2c^3 + s^3c^2 + c^5 = s^5 + s^2c^2(s+c) + c^5 = s^5 + 3 + c^5 = 126$

So $s^5 + c^5 = 126 - 3 = 123$

Since $\dfrac{\sec^{10} x + 1}{\sec^5 x} = \sec^5 x + \dfrac{1}{\sec^5 x} = \sec^5 x + \cos^5 x$, its value is 123

3. Since $\log_{1/x}(x) = \log_{1/x}\left(\frac{1}{x}\right)^{-1} = -1\left(\log_{1/x}\left(\frac{1}{x}\right)\right) = -1$ and $\log_x(x^5) = 5\log_x x = 5$

and $\log_{\sqrt{x}}\left(x^{x^2}\right) = \log_{\sqrt{x}}\left(\sqrt{x}\right)^{2x^2} = 2x^2\left(\log_{\sqrt{x}}\sqrt{x}\right) = 2x^2$

So our equation is $-1 + 5 = 2x^2$ so $2x^2 = 4$, or $x^2 = 2$ thus since $x > 0$, $x = \sqrt{2}$

4. $\log\sin x + \log\cos x + \log\tan x = \log(\sin x)(\cos x)(\tan x) = \log\sin^2 x$

$$= \log\left(1 - \cos^2 x\right) = \log\left(1 - \left(\frac{3}{\sqrt{10}}\right)^2\right) = \log\left(1 - \frac{9}{10}\right) = \log(.1) = -1$$

5. $\sec\theta = \log_{\sqrt{3}}\left(\dfrac{1}{\log_{\sqrt[3]{\pi}}(\pi)}\right) = \log_{\sqrt{3}}\left(\log_{\sqrt[3]{\pi}}(\pi)\right)^{-1} = -1\log_{\sqrt{3}}\left(\log_{\sqrt[3]{\pi}}\left(\sqrt[3]{\pi}\right)^3\right)$

$$= -1\log_{\sqrt{3}}\left(3\log_{\sqrt[3]{\pi}}\left(\sqrt[3]{\pi}\right)\right) = -1\log_{\sqrt{3}}(3) = -1\log_{\sqrt{3}}\left(\sqrt{3}\right)^2 = -2\log_{\sqrt{3}}\left(\sqrt{3}\right) = -2$$

Since $\sec\theta = -2$, $\cos\theta = -\frac{1}{2}$

So now the smallest $\theta > 0$, is $\theta = 2\pi/3$